european social fund in the czech republic
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

MATHEMATICS

exam

1	2	3	4	5	6	$\sum$

Name: .....

Date: .....

1. **[8p]** Solve the system

$$\begin{aligned} x_1 - x_2 + x_3 + 2x_4 &= 1 \\ x_1 - 2x_2 - x_3 + 2x_4 &= 1 \\ 2x_1 + 3x_3 + x_4 &= 2 \\ x_1 + x_2 + 3x_3 &= 1 \end{aligned}$$

   (a) Find the rank of the coefficient and the augmented matrix and determine how many solutions the system has.

   (b) Find the solution of the system (if exists any).

2. **[7p]** Inverse matrix

   (a) Write definition of the inverse of a matrix.

   (b) Find the inverse matrix to the matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix}.$$

   (c) Suppose that A is an $n \times n$ invertible matrix and $\vec{b}$ is an n-dimensional column vector. How many solutions are there to the system of linear equations $A\vec{x} = \vec{b}$? Write formula for the solution (using inverse of A).

3. **[10p]** Find the integrals

   (a) $\displaystyle\int \frac{x^3+1}{x^4}\,\mathrm{d}x$

   (b) $\displaystyle\int \operatorname{tg} x\,\mathrm{d}x$

   (c) $\displaystyle\int x \sin x\,\mathrm{d}x$

   (d) $\displaystyle\int_0^1 (2x^3 - x^2 + 2)\,\mathrm{d}x$

4. **[11p]** Investigate the function

$$y = \frac{x}{(x-1)^2}$$

   and sketch the graph.

5. **[6p]** Find derivatives of the following functions:

   (a) $y = \dfrac{\operatorname{arctg} x}{\ln x}$

   (b) $y = x^2 e^x$

   (c) $y = \cos(3x + 6)$

6. **[8p]** Sketch graphs and find limits at ∞ for the following functions:

   (a) $y = \operatorname{arccotg} x$

   (b) $y = e^x$

   (c) $y = \dfrac{1}{x^2}$

   (d) $y = \ln x$

$\star$ passing is $40\% = 20$ points