

Příklady: Lineární diferenciální rovnice druhého řádu

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Homogenní rovnice

- $y'' - y' - 2y = 0$ $[y = C_1 e^{2x} + C_2 e^{-x}]$
- $y'' + 5y' + 6y = 0$ $[y = C_1 e^{-3x} + C_2 e^{-2x}]$
- $y'' - 4y' = 0$ $[y = C_1 + C_2 e^{4x}]$
- $y'' - 4y = 0$ $[y = C_1 e^{2x} + C_2 e^{-2x}]$
- $y'' + 4y = 0$ $[y = C_1 \cos 2x + C_2 \sin 2x]$
- $y'' - 4y' + 13y = 0$ $[y = C_1 e^{2x} \cos 3x + C_2 e^{2x} \sin 3x]$
- $y'' - 6y' + 9y = 0$ $[y = C_1 e^{3x} + C_2 x e^{3x}]$
- $y'' - 8y' + 17y = 0, \quad y(0) = 2, \quad y'(0) = 3$ $[y = 2e^{4x} \cos x - 5e^{4x} \sin x]$
- $y'' + 2y' + y = 0, \quad y(1) = 0, \quad y'(1) = -1$ $[y = e^{1-x}(1-x)]$

Nehomogenní rovnice

Řešte metodou neurčitých koeficientů.

- $y'' + 3y' + 2y = \frac{1}{e^{x+1}}$ $[y = C_1 e^{-x} + C_2 e^{-2x} + e^{-x-1}(x-1)]$
- $y'' - 2y' - 3y = e^{4x}$ $[y = C_1 e^{3x} + C_2 e^{-x} + \frac{1}{5} e^{4x}]$
- $y'' - 3y' + 2y = x$ $[y = C_1 e^{2x} + C_2 e^x + \frac{x}{2} + \frac{3}{4}]$
- $y'' - 5y' + 6y = x^2 e^x$ $[y = C_1 e^{3x} + C_2 e^{2x} + e^x \left(\frac{x^2}{2} + \frac{3x}{2} + \frac{7}{4} \right)]$
- $y'' + 6y' + 9y = 3$ $[y = C_1 e^{-3x} + C_2 x e^{-3x} + \frac{1}{3}]$
- $y'' + 2y' + 2y = e^{-x}$ $[y = e^{-x}(C_1 \cos x + C_2 \sin x + 1)]$

7. $y'' + 2y' + 2y = e^{-x}$ $[y = e^{-x}(C_1 \cos x + C_2 \sin x + 1)]$
8. $y'' - 3y' + 2y = 5 \sin 2x$ $[y = C_1 e^{2x} + C_2 e^x + \frac{3}{4} \cos 2x - \frac{1}{4} \sin 2x]$
9. $y'' + y = x^3$ $[y = C_1 \cos x + C_2 \sin x + x^3 - 6x]$
10. $y'' + 4y = \cos 2x$ $[y = C_1 \cos 2x + C_2 \sin 2x + \frac{1}{4} x \sin 2x]$
11. $y'' - 2y' + 1 = x e^x$ $[y = C_1 e^x + C_2 x e^x + \frac{1}{6} x^3 e^x]$

Řešte metodou variace konstant.

1. $y'' + y = \frac{1}{\cos x}$ $[y = C_1 \cos x + C_2 \sin x + x \sin x + \cos x \ln |\cos x|]$
2. $y'' - 2y' + y = \frac{e^x}{x}$ $[y = C_1 e^x + C_2 x e^x + x e^x \ln |x|]$
3. $y'' + 4y' + 4y = x^{-3} e^{-2x}$ $[y = C_1 e^{-2x} + C_2 x e^{-2x} + \frac{e^{-2x}}{2x}]$